Traffic assignment algorithms for planning a mass vehicular evacuation

Houssein Alaeddine[a,b], Kamal Serrhini[a], Emmanuel Néron[b], Mindjid Maizia[a]

[a] *Laboratoire Citères de l'université de Tours*
[b] *Laboratoire Informatique de l'université de Tours*
Tours, France

Abstract

Dynamic network flow models form a wide area in evacuation domain. Within a time horizon, they aim to meet several problems as maximizing flow (maximum dynamic flow), compliance with evacuation priorities (lexicographic maximum dynamic flow), minimizing total clearance time (quickest dynamic flow) and maximizing flow for every discrete time step (universal dynamic maximum flow). The strategy of automatic construction of paths constitutes the foundation structure of these models, in other words when a predetermined set of paths among origins-destinations must be respected then flow models do not achieve the desired goal. We develop here a traffic assignment algorithms, based on the system optimum hypothesis, which allow to observe predetermined evacuation paths with flow-dependent transit time on arcs. Also we will show in this paper the importance of compliance with evacuation priorities and routes in achieving a non-preemption building evacuation.

Keywords:
routing, evacuation, traffic assignment, optimization, linear programming, meta-heuristic

1. Introduction

Flow variation over time is applied to various problems in network such as air traffic control, production systems, evacuation, etc. Two aspects distinguish dynamic flow from a static one, first the flow on arcs may change over time and second the flow does not traverse the network instantly, but requires a certain amount of time to cross each arc. Dynamic flow problem is traditionally solved by a time-expanded network, introduced by Ford and Fulkerson [17], that contains a copy of the static network for each discrete time step $t \in \{0, 1, \ldots, T-1\}$ where T is the number of time steps (determined from an evacuation time horizon) during which the evacuation process must be executed [1]. In other words, travel time on each arc is converted to time steps and no flow is allowed to enter the network unless its last particle (e.g. vehicle) reaches its destination at the latest at T. Ford-Fulkerson assumed journey times and capacities of arcs as constants.

Given a static graph $G = (N, A)$, a time-expanded graph $G_{T-1} = (N_{T-1}, A_{T-1})$ is built so that $N_{T-1} = \{j(t) : j \in N, t \in \{0, 1, \ldots, T-1\}\}$ and $A_{T-1}\{(i(t), j(t+\tau_{i,j})), t \in \{0, \ldots, T-1-\tau_{i,j}\}\}$ respectively represent the set of nodes and the set of arcs, with $\tau_{i,j}$ indicating the travel time on arc (i, j). Fleischer [14] showed that dynamic maximum flow problem during T is a classical problem of maximum flow from source $s(0)$ to destination $p(T-1)$. Time-expanded graph has T copies of each source node and each destination node. Moreover this network of multiple sources-destinations may be converted to a single source-destination network. Time-expanded networks are useful here : the journey time on arcs remains implicit and problems that are traditionally solved in a static network can also be solved in a dynamic network.

The capacity of arc (i, j) in time-expanded network is expressed in the maximal number of units which can pass through this arc by unit of time (e.g. 20 veh/min). The time period between two time steps t and $t+1$ is given by δ. For example, for δ = 15 seconds and an arc (i, j) with $\tau_{i,j} = 2$, each unit of a flow crossing (i, j) will take 30 seconds. The constant travel time on arc (i, j) is computed with a flow equal to the capacity of this arc (flow-independent time). The number of time steps T is computed by dividing the given horizon time by δ. A too low value of δ results in a huge graph with fractional capacities of arcs (for instance, $u_{i,j} = 0.38888$veh/sec), and it is not practical to work with such a network. Similarly, a very high value of δ has the drawback to result in a graph with fractional travel times (for example, $\tau_{i,j} = \frac{3min}{1h}$). Such a system is inefficient because it no longer depends on the properties of flow. Therefore, the value of δ is a compromise between model realism and model complexity [22]. The other drawback of the time-expanded network lies in the discrete nature of the time step. The construction of time-expanded network is based on a fixed number of time units T computed from the evacuation time horizon (as mentioned above). Each node in the static graph is copied T times in the time-expanded network, however arcs do not comply with this principle but are copied based on travel time through the arcs (expressed in number of time units). In other words, this type of network is no longer valid if travel time on arcs depends on flow.

In this type of network, as we mentioned previously, $\tau_{i,j}$ is the maximum travel time (upper bound) on the arc (i, j) i.e. in the presence of the maximum number of vehicles (equal to the capacity). Kohler et al. [27] varied

Email addresses: houssein.alaeddine@hotmail.fr (Houssein Alaeddine), kamal.serrhini@univ-tours.fr (Kamal Serrhini), emmanuel.neron@univ-tours.fr (Emmanuel Néron), mindjid.maizia@univ-tours.fr (Mindjid Maizia)

1. A period of time is the real time between two successive time steps. For example, for a time horizon equals to 18 seconds and a time period equals to 3 seconds, the value of T is equal to $6=\frac{18}{3}$. Concretely, steps 0,1,2,3,4,5 and 6 respectively correspond to the lower bounds of the time intervals [0,3), [3,6), [6,9), [9,12), [12,15), [15,18) and [18,21)

journey times on arcs with flow in order to overcome this drawback. However, this variability of time is discrete (linear function per 3 pieces or intervals), i.e, the travel time on arc can relate to only three distinct values. Increase the number of pieces leads to an impracticable too large graph. Finally, the complexity of dynamic flow algorithms based on time-expanded network is pseudo-polynomial i.e. the execution time of these algorithms is a polynomial function of the number of time periods T [25]. To remove the logical obstacles of the time-expanded network approach, Ford-Fulkeron introduced the flow temporally repeated based on the maximum flow minimum cost algorithm in the static network. Let F a feasible static flow in G = (N,A) and $f^1, f^2, \ldots, f^r$ the flows on paths $p^1, p^2, \ldots, p^r$ determined by a path decomposition algorithm (see [11]), where $\sum_{i=1}^{r} f^i = \mathrm{F}$. The travel time on path p^i is given by $\tau(p^i) = \sum_{a \in p^i} \tau_a$ and the principle of flow temporally repeated is to inject f^i units of flow on p^i at each time step $t \in \{0, 1, \ldots, \mathrm{T} - 1 - \tau(p^i)\}$.

It's well known as mentioned previously, that any multi-sources problem can be transformed into a single source problem by creating a super-source. Nevertheless this transformation is not feasible in the flow temporally repeated approach. Firstly, the computed amount of flow from some sources may be equal zero (network saturated), and secondly, a source with a small number of vehicles does not require the entire time horizon to evacuate its vehicles. Many authors addressed the flows over time in network offering different efficient approachs for different problems (dynamic maximum flow [17, 16, 14], universal dynamic maximum flow [18, 25, 29, 38, 25, 12, 31, 15], quickest dynamic flows [13, 10, 8, 25], lexicographic dynamic maximum flow [29, 22, 23, 25]). The foundation structure of these models is based on the automatic construction of paths, therefore if a predetermined set of paths among origins-destinations must be observed then flow models do not achieve the desired aim.

In contrast to flow models, traffic assignment systems deal in a more realistic manner with traffic in network. Generally they are divided into two different systems : the system optimum and the user or nash equilibrium [37] [34] [7] [33]. Widely studied in the literature dedicated to transport and its applications, these two types of systems have also been used for managing traffic during a massive emergency evacuation. These models predict, optimize and plan traffic operations, where evacuees behavior in terms of choice of routes is taken into account by specific constraints. Dynamic traffic assignment models can be used to estimate the network load over time depending on its request. They are characterized by two interdependent elements : route choice and dynamic network loading. Route selection methods predetermine the evolution of flow on the roads, while dynamic network loading describes the outflow and the spread of flow through the network [24] [26] [6] [32]. During the evacuation, selfish drivers attempt to switch from a road to another in order to minimize their travel times. The user equilibrium system, first principle of Wardrop [37], states that travel times on used routes connecting an origin-destination pair are equal and minimal, i.e., no driver can unilaterally reduce his/her travel time by shifting to another route (reaching steady state). As it's well known, this system used by transport planners for estimating and preventing network congestion does not minimize the total clearance time. In contrast, Wardrop's second principle states that travelers cooperate with each other in order to minimize the total travel time and to ensure a traffic without congestion (System Optimum). Drivers in this system are informed about the paths to be taken. Obviously this is not a behaviorally realistic system, but it can be a useful traffic management tool for transport planners in order to minimize the total travel cost and therefore achieve an optimum social equilibrium [30].

Stepanov and Smith [35] developed a traffic assignment model for the evacuation problem, which minimizes the total clearance time, total distance and traffic-jams. Given a set of candidate paths between each pair origin-destination, the integer linear model aims, under the objectives mentioned previously, to find one egress route for each origin. Bretschneider and Kimms [9] considered a mixed-integer heuristic approach, based on the time-expanded network, for reorganizing traffic during an evacuation process. The main objective in that model is to allow people to leave the dangerous area in a minimum time. Lim et al. [28] presented a flow scheduling/optimization model for maximizing and scheduling the flow entering the evacuation network. The transit times on road links are estimated based on the peak rush hour travel time on a major highway.

After synthesizing the literature about the modeling of an evacuation problem, we present in this paper a model, based on the system optimum hypothesis, for the evacuation of population exposed to natural disasters. A linear program and a meta-heuristic are implemented to compute an evacuation plan with a minimum total clearance time. We study here the dynamic traffic assignment problem from scheduling side where at each time slot θ we evacuate by priority order the maximum number of vehicles from each building not yet completely evacuated. Predetermined paths among buildings and shelters are observed and a traffic model (Greenshield's model[21], polynomial model [5], etc.) is adopted for computing the flow-dependent time on arcs. We assume, in order to simplify this paper, the existence of a vehicles pursuit model (see [1]) which avoids the occurrence of overlap in the evacuation network. Indeed this overlap between successive time intervals arises as a result of the variation of sources of flows over time. That model which traces the paths of vehicles aims to compute the earliest departure times of buildings while avoiding any traffic-jams to occur in the network. The total duration of the discharge is provided by our model [2] and we will show in this paper the importance of compliance with evacuation priorities and routes in order to achieve a non-preemption building evacuation.

2. Graph construction

Let us consider **B** a priority list of n buildings to be evacuated towards a set of m shelters **S** via a transportation network $\mathscr{G} = (\mathscr{N}, \mathscr{A})$ established for this purpose.

2. This paper is a part of STOM (Spatio-Temporel Optimization Model for the evacuation of people exposed to natural disasters) model developed in ACCELL project funded by Région Centre and FEDER, France

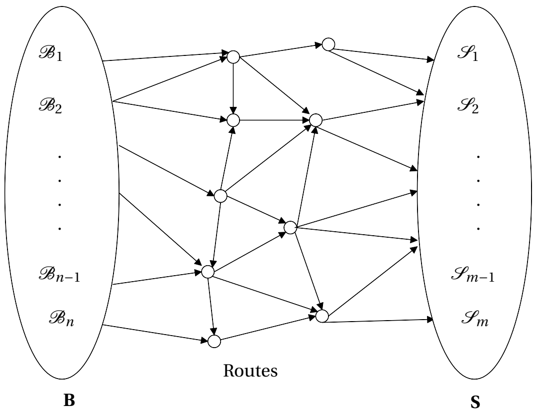


FIGURE 1: Evacuation network

Each building $\mathcal{B}_i, i \in \{1,\dots,n\}$ is defined by a number of vehicles v_i to be evacuated. Each shelter $\mathcal{S}_j, j \in \{1,\dots,m\}$ is defined by its capacity of vehicles c_j. We associate one or more shelters to each building $\mathcal{B}_i$ and we denote by $\mathcal{H}_i$ the set of these shelters. A set of K-best paths $\mathcal{K}_{i,j}$ is considered computed between each building $\mathcal{B}_i$ and each shelter $\mathcal{S}_j \in \mathcal{H}_i$. We denote by $v_{i,j}$ and $\eta_{i,j}$ successively, the number of vehicles to be evacuated and the dimension of the set of paths $\mathcal{K}_{i,j}$ from $\mathcal{B}_i$ to $\mathcal{S}_j \in \mathcal{H}_i$. Finally the transportation network $\mathcal{G} = (\mathcal{N},\mathcal{A})$ among buildings and shelters is established. We define $\hat{\phi}_l, l \in$ A, the capacity of the road link l expressed in vehicles per unit of time (veh/h, etc.). This capacity is computed by a traffic model based on free-flow speed, maximal density and number of lanes [19] [5].

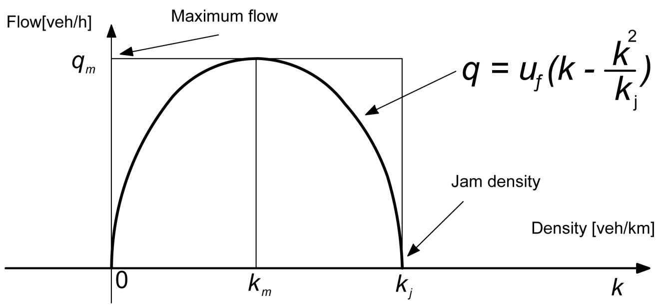


FIGURE 2: Relation between speed and density [36] : Greenshield's traffic stream model [21]

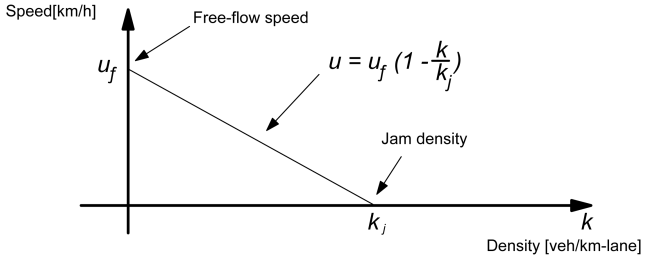


FIGURE 3: Relation between flow and density [36] : Greenshield's traffic stream model [21]

We represent the k-th-path of dimension q connecting $\mathcal{B}_i$ to $\mathcal{S}_j$ by the vector of arcs $\mathrm{Arc}^k_{i,j}$=$(e,\dots l,\dots y) \in \mathcal{A}^q$. Since the graph $\mathcal{G}$ corresponds to specific paths, which have been predetermined, connecting buildings to shelters associated, the next equality can be demonstrated :

$$\cup_{i=1}^{n} \cup_{\mathcal{S}_j \in \mathcal{H}_i} \cup_{k=1}^{\eta_{i,j}} \mathrm{Arc}^k_{i,j} = \mathcal{A} \tag{1}$$

We define a new graph $\mathcal{G}^* = (\mathcal{N}^*,\mathcal{A}^*)$ extended from $\mathcal{G}$ and which includes in addition to $\mathcal{N}$ and $\mathcal{A}$, two virtual nodes $\{s,p\}$, a sub-set of virtual nodes **M** of dimension $\sum_{i=1}^{n} |\mathcal{H}_i|$ including one node $\mathcal{M}_{j,i}$ by each safety point $\mathcal{S}_j$ associated to each building $\mathcal{B}_i$, a sub-set of virtual arcs between s and **M** which represent the maximum numbers of vehicles that may be evacuated from buildings to safety points, a sub-set of arcs of dimension $\eta_{i,j}$ between each node $\mathcal{M}_{j,i} \in$ **M** and $\mathcal{B}_i$, and finally a sub-set of virtual arcs among shelters and p which represent the capacities of shelters in terms of vehicles.

The sub-set of arcs between s and **M** is built such that each shelter $\mathcal{S}_j \in \mathcal{H}_i$ associated to $\mathcal{B}_i$, $i \in \{1,\dots,n\}$ has a virtual arc, this is the arc between s and $\mathcal{M}_{j,i}$. The weight (capacity) given for the arc $(s,\mathcal{M}_{j,i})$ is the maximum number that may be evacuated from building $\mathcal{B}_i$ to shelter $\mathcal{S}_j$ associated. This weight is the minimum value between $r_{i,j}$, the remaining vehicles to be evacuated from $\mathcal{B}_i$ to $\mathcal{S}_j$, and $\hat{\mathcal{K}}_{i,j}$, the capacity of paths connecting $\mathcal{B}_i$ to $\mathcal{S}_j$.

In the new graph $\mathcal{G}^*$, **M** is necessary firstly, to define the maximum number of vehicles that may be evacuated from buildings at each time slot, and secondly, to know the number of vehicles evacuated on paths connecting buildings to shelters associated. The figure 4 illustrates the new graph $\mathcal{G}^*$. It should be noted that in this figure the weights on arcs $(s,\mathcal{M}_{j,i})$ correspond to capacity, while those on $(\mathcal{M}_{j,i},\mathcal{B}_i)$ correspond to flow.

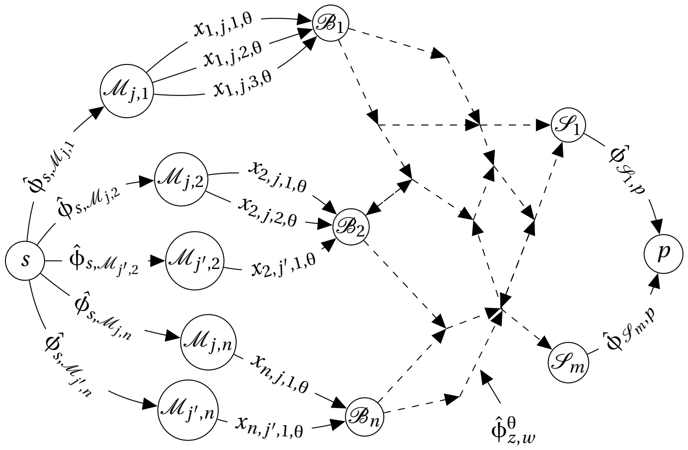


FIGURE 4: Graph $\mathcal{G}^* = (\mathcal{N}^*,\mathcal{A}^*)$

3. Problem formulation

An evacuation plan aims to fix for each building its departure times and the paths for reaching its destinations. The computation of this plan is based on two pairs of concepts (time slot, flow) and (time interval, flow rate). The first pair focuses on a chronological evacuation according to a priorities list of buildings. In a complementary manner, the second pair is based on computing departure and arrival times in order to avoid any overlap to occur in the network among successive time slots.

We are interested here only to present the evacuation scheduling optimization system (time slot, flow) which is mainly subjected to the evacuation priorities of buildings, the destinations of evacuation and the K-best paths connecting each building to each shelter associated.

Notation :

- i : an index for buildings (population sources) $\mathscr{B}_i$
- v_i : number of vehicles of $\mathscr{B}_i$
- j : an index for shelters (destinations) $\mathscr{S}_j$
- c_j : capacity of $\mathscr{S}_j$
- k : an index for k-th best path
- θ : an index for time slot
- $\hat{\phi}_l^\theta$: capacity of arc l at time slot θ
- l : road link (arc)
- ϕ_l^θ : flow on arc l at time slot θ
- $\mathscr{M}_{j,i} : \mathscr{B}_i \to \mathscr{S}_j$ ($\mathscr{B}_i$ is associated to $\mathscr{S}_j$)

Decision variables :

- $x_{i,j,k,\theta}$: number of vehicles evacuated from building i to safety point j using the k-th path connecting $\mathscr{B}_i$ and $\mathscr{S}_j$ at time slot θ
- $\phi_{v,z}^\theta$: number of vehicles transiting on arc (v,z) at time slot θ

Auxiliary variables :

- $e_{i,\theta} = \begin{cases} 1 & \text{if } \mathscr{B}_i \text{ is effectively evacuated, completely} \\ & \text{or partially, at time slot } \theta \\ 0 & \text{otherwise} \end{cases}$

Parameters :

- $\beta_{i,j} = \begin{cases} 1 & \text{if } \mathscr{B}_i \text{ is connected to } \mathscr{S}_j \\ 0 & \text{otherwise} \end{cases}$
- $\alpha_{i,j,k,l} = \begin{cases} 1 & \text{if } l \in Arc_{i,j}^k \\ 0 & \text{otherwise} \end{cases}$

4. Evacuation model

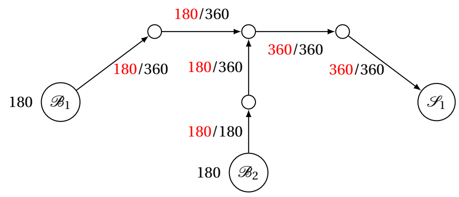
During a flooding event, buildings that are located in dangerous areas have different level of hazard i.e. there are zones that must be evacuated before another ones and so on. In addition, being the transport network does not enable a simultaneously evacuation of all people we evacuate buildings by time slots according to a established priorities list. Moreover, the evacuation of each building must be of minimum duration and cannot be interrupted (non-preemption). The problem addressed here is in the domain of optimal routing problems where the aim goal is to minimize the total clearance time. We approach this problem via a multi-objective scheduling optimization model.

The strategy applied in this model to meet the objectives stated above is to maximize lexicographically the flow outgoing from sources. Moreover as it will be shown, the compliance with predetermined paths will contribute to achieve a sequence of evacuation time slots of each building.

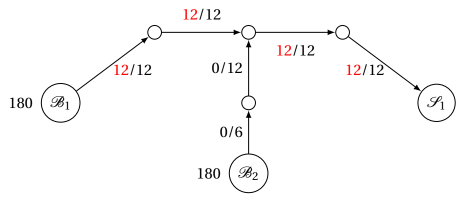
Capacity on roads depends on slot time unit i.e. how many vehicles can cross each road by time unit (for instance 15min). This capacity, as we mentioned previously, is computed by a traffic model based on the density, the free-flow speed and the number of lanes of each road.

In dynamic flow problems, discretization plays a pivotal role in the evacuation modeling. A very small time slot (for example, minute or second) increases the accuracy of the model but enlarges the size of graph and thus the computational complexity of the solution algorithms [22]. The difference results between a small and a big time slot is the fact that transit times on roads that are computed in terms of time slots don't depend on flow. Moreover vehicles flow rate on a path cannot be depending on flow rate of minimum arc due to sharing arcs among paths from different sources. Therefore vehicles need a period equal to slot time unit to exit each source in order to avoid any congestion on the network. For example in the temporally repeated flow, for a time unit of 15 minutes, the evacuation of a building with 10 vehicles requires 15 minutes plus the flow-independent journey time of the last vehicle.

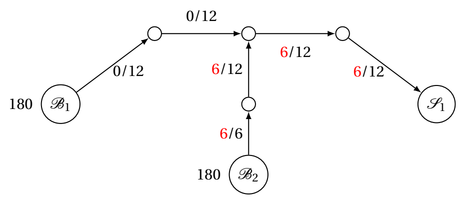
The accuracy of a model does not necessarily increase with a small time slot. Here we give an example showing that slot of half hour is preferable than that of one minute in terms of clearance time. The figure 5 makes a comparison between time step (half hour) and time step (minute). We remark that in time step half hour, vehicles of $\mathscr{B}_1$ and $\mathscr{B}_2$ can be evacuated in 30 minutes. While in time step minute, $\mathscr{B}_1$ needs 15 minutes and then $\mathscr{B}_2$ takes 30 minutes. It should be noted that if $\mathscr{B}_2$ has a higher priority than $\mathscr{B}_1$ or if they have both the same priority then the total evacuation times are identical for the two cases (half hour and minute). In this comparison the transit times on arcs are not considered.



a: time slot (half hour) : $\mathscr{B}_1$ and $\mathscr{B}_2$ become empty 30 minutes after the start of evacuation



b: time slot (minute) : $\mathscr{B}_1$ becomes empty 15 minutes after the start of evacuation



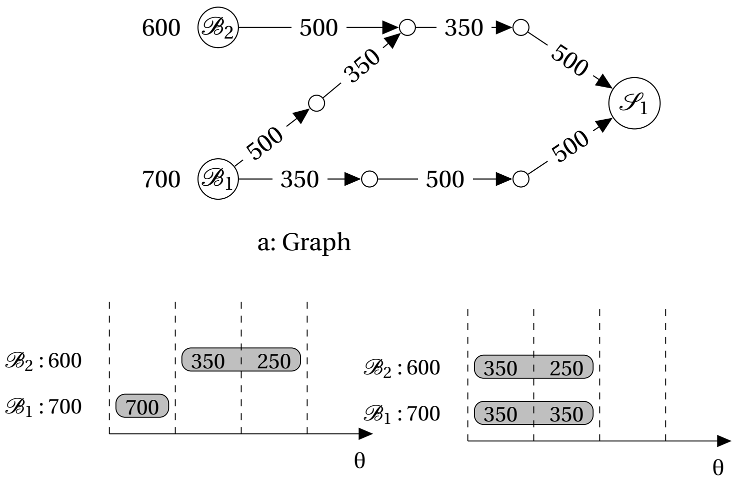
c: time slot (minute) : $\mathscr{B}_2$ becomes empty 30 minutes after the start time of the last vehicle from $\mathscr{B}_1$

FIGURE 5: Preference of time step half hour relative to time step minute

The difference between slot of half hour and that of one minute is due to the fact that parallel evacuation ensuring in the case of slot of half hour is missed in the other case. This parallelism can be kept in the second case if we start evacuating $\mathscr{B}_2$ firstly, where as we see that the capacity of path from $\mathscr{B}_1$ is twice that of path from $\mathscr{B}_2$.

The number of paths of buildings plays an essential role in the evacuation. Indeed, the total number of slots decreases when the number of available paths is high. To further reduce the number of slots, and in the case of buildings with the same level of priority, the model first evacuates buildings with few roads. Figure 6 shows the importance of this strategy in terms of reducing the number of slots and adding an element of fairness to the flow assignment model.

In this figure we consider the priorities list $\{\mathscr{B}_1, \mathscr{B}_2\}$. While $\mathscr{B}_1$, with 700 vehicles to evacuate, has two paths to shelter $\mathscr{S}_1$, the building $\mathscr{B}_2$ has only one way to evacuate its 600 vehicles (part a, figure 6). We assume $\mathscr{B}_1$ and $\mathscr{B}_2$ have the same evacuation priority. The part (b) shows the evacuation scheduling (3 time slots) of $\mathscr{B}_1$ and $\mathscr{B}_2$ regardless of the number of paths, while part (c) illustrates not only the parallel evacuation of two buildings, but also the reduction of the number of time slots.



a: Graph

b: $\mathscr{B}_1$ puis $\mathscr{B}_2$

c: $\mathscr{B}_2$ puis $\mathscr{B}_1$

FIGURE 6: Number of paths ; weight on arcs : capacity

5. Lexicographic maximum traffic assignment

The lexicographical maximum flow algorithm presented here is a slightly modified version of one developed in a previous paper [2] using the push-relabel algorithm [3][20]. Being the difficulty to integrate the constraint of compliance with predetermined paths, as we have shown in that paper, a linear program subjected to this constraint is developed here. This program is an alternative solution of the push-relabel algorithm in the lexicographic maximum flow algorithm. It should be noted that the linear program execution time is faster than that of push-relabel implemented in Boost library [4].

The principle of the lexicographic maximum flow algorithm is to send at each time slot by order of priorities, the maximal number of vehicles from each building not yet completely evacuated.

In order to simplify the algorithms developed in this paper, we assume that each building is associated to one and only one shelter. This assumption does not eliminate the possibility for several shelters, but if a building $\mathscr{B}_i$ must be evacuated to several shelters ($|\mathscr{H}_i| > 1$), then this building will be divided into $|\mathscr{H}_i|$ buildings $\{\mathscr{B}_{i_1}, \ldots, \mathscr{B}_{i_{|\mathscr{H}_i|}}\}$.

Given a sub-set of p buildings $\{\mathscr{B}'_1, \ldots, \mathscr{B}'_p\} \subseteq \{\mathscr{B}_1, \ldots, \mathscr{B}_n\}, 1 \leq p \leq n$, the strategy of the lexicographic maximum flow algorithm is based on the determination of the lexicographic maximal flow (Lex-Max) from the building $\mathscr{B}'_p$. Note that building $\mathscr{B}'_p$ corresponds to a building $\mathscr{B}_i \in \{\mathscr{B}_1, \ldots, \mathscr{B}_n\}$. Initially equal to the minimum between $r_{i,j}$ and $\hat{\mathscr{K}}_{i,j}$, Lex-Max($\mathscr{B}'_p$) decreases in the event of impact on the lexicographic maximal flows from previous buildings $\mathscr{B}'_q$ (highest priorities, $\forall\ 1 \leq q < p$) determined at previous iterations. The linear program (CPLEX) described in the following is called in each iteration p at each time slot θ with a number of buildings p increased by one unit. Note that at first time slot the number of iterations is n and $\mathscr{B}'_1 \equiv \mathscr{B}_1$, $\mathscr{B}'_2 \equiv \mathscr{B}_2, \ldots, \mathscr{B}'_{n-1} \equiv \mathscr{B}_{n-1}$, $\mathscr{B}'_n \equiv \mathscr{B}_n$.

We define the graph $\mathscr{G}^*_\theta$ which corresponds only to the buildings not yet completely evacuated. At first time slot, $\mathscr{G}^*_1 = \mathscr{G}^*$.

Moreover, we define the sub-graph $\mathscr{G}^*_{\theta,p} = \{\cup_{q=1}^{p} \cup_{k=1}^{\eta_{i,j}} Arc^k_{i,j}\ /\ \beta_{i,j} = 1\}$ which corresponds to the network connecting only buildings $\mathscr{B}'_q$ ($\forall\ 1 \leq q \leq p$) to their destinations. Note that $\mathscr{B}_i \in \{\mathscr{B}_1, \ldots, \mathscr{B}_n\}$ corresponds to the building $\mathscr{B}'_q$.

At each step p in each time slot θ, we build $\mathscr{G}^*_{\theta,p}$ tagging the capacity of the arc $(s, \mathscr{M}_{j,i})$ with the maximum number of vehicles that may be evacuated from $\mathscr{B}_i$ to $\mathscr{S}_j$ ($\mathscr{B}_i$ corresponds here to $\mathscr{B}'_p$). As we mentioned above, this maximum number represented in the graph (see figure 4) by $\hat{\phi}_{s,\mathscr{M}_{j,i}}$ is equal to the minimum value between $r_{i,j}$ et $\hat{\mathscr{K}}_{i,j}$. Also the maximum number of vehicles that may be evacuated through $Arc^k_{i,j}$ (and which is represented by $\hat{\phi}^k_{\mathscr{M}_{j,i},\mathscr{B}_i}$ in the graph) is equal to the minimum value between $r_{i,j}$ and $Min_{\hat{\phi}} Arc^k_{i,j}$ the capacity of k-th path. Besides, the capacity of each arc $(s, \mathscr{M}_{j,i})$ ($\mathscr{B}_i$ corresponds to $\mathscr{B}'_q$, $\forall 1 \leq q < p$) is equal to Lex-Max($\mathscr{B}'_q$) determined at step q.

We initialize Lex-Max($\mathscr{B}'_p$) to $\hat{\phi}_{s,\mathscr{M}_{j,i}}$ ($\mathscr{B}_i$ corresponds here to $\mathscr{B}'_p$), then we decrease this value if the evacuation priorities may be violated. This potentially violation is detected when the sum of flow assigned from all buildings in graph $\mathscr{G}^*_{\theta,p}$ is lower than the sum of the lexicographical maximal flows of these buildings, which is translated by the next equation :

$$\sum_{q=1}^{p} \phi_{s,\mathscr{M}_{j,i}} < \sum_{q=1}^{p} \hat{\phi}_{s,\mathscr{M}_{j,i}} \tag{2}$$

where $\mathscr{B}_i$ corresponds to $\mathscr{B}'_q$, $\forall 1 \leq q \leq p$

In this case, Lex-Max($\mathscr{B}'_p$) decreases and takes a new value : this is the difference between the flow assigned from all buildings and the lexicographical maximal flows of previous

3. An efficient approach for computing maximum flow in a graph
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buildings only.

$$\hat{\phi}_{s,\mathscr{M}_{j,i'}} = \phi_{s,\mathscr{M}_{j,i'}} + \sum_{q=1}^{p-1} \phi_{s,\mathscr{M}_{j,i}} - \sum_{q=1}^{p-1} \hat{\phi}_{s,\mathscr{M}_{j,i}} \tag{3}$$

where $\mathscr{B}_i$ corresponds to $\mathscr{B}'_q$, $\forall 1 \leq q \leq p-1$ and $\mathscr{B}_{i'}$ corresponds to $\mathscr{B}'_p$.

The linear program (CPLEX) is presented in the following. The objective function of this model is to maximize flow from $\{\mathscr{B}'_1, \ldots, \mathscr{B}'_p\}$ subjected to three constraints. The first constraint for ensuring the conservation of flows, the second one for respecting the capacity of each arc and the last constraint for complying with predetermined evacuation routes.

Algorithm 1 Lexicographic maximum flow algorithm

1: **procedure** FIND LEXICOGRAPHIC MAXIMUM FLOW
2: **repeat**
3: Open a new time slot θ
4: $r = 0$ ▷ Number of iterations
5: $Sum = 0$
6: Let $\mathscr{Q}$ the set of buildings not yet completely evacuated
7: **repeat**
8: $r = r + 1$
9: Take the priority building $\mathscr{B}_i$ from $\mathscr{Q}$ ▷ $\mathscr{B}'_r = \mathscr{B}_i$
10: $\mathscr{P}.add(\mathscr{B}_i)$ ▷ $\mathscr{S}_j \in \mathbf{S}/\beta_{i,j} = 1$
11: $\hat{\phi}_{s,\mathscr{M}_{j,i}} = Min\{r_{i,j}, \hat{\mathscr{K}}_{i,j}\}$ ▷ Number of vehicles to be evacuated from $\mathscr{B}_i$
12: $\hat{\phi}^k_{\mathscr{M}_{j,i},\mathscr{B}_i} = Min\{r_{i,j}, Min_{\hat{\phi}} Arc^k_{i,j}\}, \forall 1 \leq k \leq \eta_{i,j}$
13: $\hat{\phi}_{\mathscr{S}_{j'},p} = \sum_{q=1}^{r} \hat{\phi}_{s,\mathscr{M}_{j',i'}}, \forall \mathscr{S}_{j'} \in \mathbf{S}/\beta_{i',j'} = 1$ ▷ Shelters capacity, $\mathscr{B}_{i'} \equiv \mathscr{B}'_q$
14: $Sum += \hat{\phi}_{s,\mathscr{M}_{j,i}}$
15: $Flow = CPLEX(\mathscr{P})$ ▷ Compute the maximal flow from $\{\mathscr{B}'_1, \ldots, \mathscr{B}'_r\}$
16: **if** $Flow < Sum$ **then** ▷ Priority may not have been respected
17: $Sum -= \hat{\phi}_{s,\mathscr{M}_{j,i}}$
18: $\hat{\phi}_{s,\mathscr{M}_{j,i}} = Flow - Sum$ ▷ Compute a new Lex-Max from $\mathscr{B}_i \equiv \mathscr{B}'_r$
19: $Sum += \hat{\phi}_{s,\mathscr{M}_{j,i}}$
20: **end if**
21: $\mathscr{Q}.remove(\mathscr{B}_i)$
22: **until** $\mathscr{Q} = \emptyset$
23: **if** $\hat{\phi}_{s,\mathscr{M}_{j,i}} < Min\{r_{i,j}, \hat{\mathscr{K}}_{i,j}\}$ **then** ▷ If initial Lex-Max($\mathscr{B}'_r$) is decreased
24: $Flow = CPLEX(\mathscr{P})$ ▷ Definitive maximal flow respecting priorities at slot θ
25: **end if**
26: **repeat**
27: $x_{i,j,k,\theta} = \hat{\phi}^k_{\mathscr{M}_{j,i},\mathscr{B}_i}, \forall k \in \{1, \ldots, \eta_{i,j}\}$
28: $r_{i,j} = r_{i,j} - \hat{\phi}_{s,\mathscr{M}_{j,i}}$ ▷ Number of residual vehicles of $\mathscr{B}_i$
29: $\mathscr{P}.remove(\mathscr{B}_i)$
30: **until** $\mathscr{P} = \emptyset$
31: **until** all vehicles from all buildings are already evacuated
32: **end procedure**

$$\begin{cases} \text{Max}_{\mathbf{x}} \sum_{q=1}^{p} \sum_{\mathscr{S}_j \in \mathscr{H}_i} \sum_{k=1}^{\eta_{i,j}} x_{i,j,k,\theta} \\ s.t. \\ \sum_{l \in pred(z)} \Phi^{\theta}_{l} = \sum_{l' \in succ(z)} \Phi^{\theta}_{l'} & , \forall z \in \mathscr{N}^*_{\theta,p} \backslash \{s,p\} \\ \Phi^{\theta}_{l} \leq \hat{\Phi}_l & , \forall l \in \mathscr{A}^*_{\theta,p} \\ \Phi^{\theta}_{l} = \sum_{i=1}^{r} \sum_{\mathscr{S}_j \in \mathscr{H}_i} \sum_{k=1}^{\eta_{i,j}} x_{i,j,k,\theta} \cdot \alpha_{i,j,k,l} & , \forall l \in \mathscr{A}_{\theta,p} \\ x_{i,j,k,\theta} \in \mathbb{N} & , \forall 1 \leq q \leq p, \beta_{i,j} = 1, \forall k \end{cases}$$

where $\mathscr{B}_i$ corresponds to $\mathscr{B}'_q$, $\forall 1 \leq q \leq p$

The figure 7 provides an example of an evacuation network comprising five buildings to be evacuated. The established priorities list of these buildings is $\{\mathscr{B}_2, \mathscr{B}_1, \mathscr{B}_4, \mathscr{B}_5, \mathscr{B}_3\}$. Note that the two buildings $\{\mathscr{B}_4, \mathscr{B}_5\}$ constitutes in reality the same building, but as this latter must be evacuated to two shelters then we divided it here to two buildings. The input data of this example is illustrated in the figure 8. The K-minimum paths ($k = 2$) between each origin-destination are computed by a multicriteria algorithm (transit time, capacity). However all or a set of these paths can be in some cases provided by the decisions makers.

We are interested here to show the difference in results between the lexicographic maximum flow algorithm coupled with CPLEX and the same algorithm coupled with push-relabel. The disparity between these two results lies in the compliance level with predetermined paths. While this constraint is completely ensured in the linear program (see above, third constraint), the non-respect of predetermined paths is always existing in push-relabel despite the using of two strategies for limiting this problem : the first one is to restrict the capacities of shelters at each iteration (line 13, algorithm 1) and the second one involving the construction of a sub-graph at each iteration. It should be noted that the observation of predetermined paths automatically leads to the respect of destinations.

The table 4 shows the flow on arcs at first time slot injected by CPLEX, while the table 5 is an illustration of the allocation of flow carried out by push-relabel. This second table indicating the residual capacity of each arc at the first time slot proves the construction of new paths in order to maximize the flow in the network. During the evacuation of $\mathscr{B}_4$, push-relabel has built a new path $\{a_{17}, a_{18}, a_{19}, a_{20}, a_{11}, a_{12}\}$ allowing to evacuate 100 vehicles from this building. It should be noted that if capacities on shelters are relaxed, push-relabel algorithm would have built the path $\{a_{13}, a_{14}, a_{19}, a_{20}, a_{11}, a_{12}\}$ to evacuate 200 vehicles from $\mathscr{B}_3$ in addition to 100 vehicles evacuated from this building by its minimal path $\{a_{13}, a_{14}, a_{15}, a_{16}\}$ (see figure 8).

l	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
$\hat{\phi}_l$	500	350	350	700	350	500	500	500	350	350	700	700	350	500	600	600	600	600	500	350
$\mathscr{B}_2 \rightarrow \mathscr{S}_1$								150	0	0	350	350								
$\mathscr{B}_2 \rightarrow \mathscr{S}_1$		200	200	550				0												
$\mathscr{B}_1 \rightarrow \mathscr{S}_1$	300	0	0	350																
$\mathscr{B}_1 \rightarrow \mathscr{S}_1$				0	0	150	150													
$\mathscr{B}_4 \rightarrow \mathscr{S}_1$			✗	✗																
$\mathscr{B}_5 \rightarrow \mathscr{S}_2$															0	0	0	0		
$\mathscr{B}_3 \rightarrow \mathscr{S}_2$															✗	✗				

TABLE 4: Assigned flow on arcs at first slot using lexicographic maximum flow algorithm coupled with CPLEX

l	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}
$\hat{\phi}_l$	500	350	350	700	350	500	500	500	350	350	700	700	350	500	600	600	600	600	500	350
$\mathscr{B}_2 \rightarrow \mathscr{S}_1$								150	0	0	350	350								
$\mathscr{B}_2 \rightarrow \mathscr{S}_1$		200	200	550				0												
$\mathscr{B}_1 \rightarrow \mathscr{S}_1$	300	0	0	350																
$\mathscr{B}_1 \rightarrow \mathscr{S}_1$				0	0	150	150													
$\mathscr{B}_4 \rightarrow \mathscr{S}_1$											250	250					500	500	400	250
$\mathscr{B}_5 \rightarrow \mathscr{S}_2$															100	100	0	0		
$\mathscr{B}_3 \rightarrow \mathscr{S}_2$													250	400	0	0				

TABLE 5: Assigned flow on arcs at first slot using lexicographic maximum flow algorithm coupled with push-relabel

The behavior of the lexicographic maximum flow

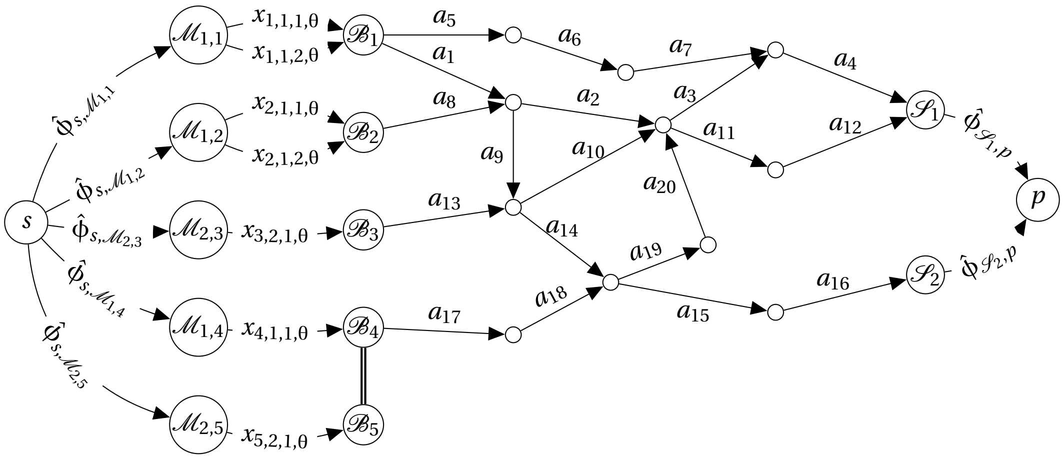

FIGURE 7: Sample evacuation network $\mathcal{G}^* = (\mathcal{N}^*, \mathcal{A}^*)$

$Arc^1_{1,1}$	$Arc^2_{1,1}$	$Arc^1_{2,1}$	$Arc^2_{2,1}$	$Arc^1_{3,2}$	$Arc^1_{4,1}$	$Arc^1_{5,2}$
a_1	a_5	a_8	a_8	a_{13}	a_{17}	a_{17}
a_2	a_6	a_2	a_9	a_{14}	a_{18}	a_{18}
a_{13}	a_7	a_3	a_{10}	a_{15}	a_{19}	a_{15}
a_4	a_4	a_4	a_{11}	a_{16}	a_{20}	a_{16}
			a_{12}		a_3	
					a_4	

TABLE 1: Paths between buildings and shelters

l	L_l (Km)	$\hat{\phi}_l$ $(veh/\frac{1}{4}h)$	l	L_l (Km)	$\hat{\phi}_l$ $(veh/\frac{1}{4}h)$
a_1	1.5	500	a_{11}	2	700
a_2	1.4	350	a_{12}	3	700
a_3	1.2	350	a_{13}	3	350
a_4	1.3	700	a_{14}	1	500
a_5	3	350	a_{15}	3	600
a_6	1.4	500	a_{16}	2	600
a_7	2	500	a_{17}	2	600
a_8	3	500	a_{18}	1.5	600
a_9	2	350	a_{19}	1	500
a_{10}	1.7	350	a_{20}	1	350

TABLE 2: Road links caracteristics

$\mathcal{B}_i$	$\mathcal{B}_1$	$\mathcal{B}_2$	$\mathcal{B}_3$	$\mathcal{B}_4$	$\mathcal{B}_5$
p_i	2250	1500	900	2100	300
v_i	750	500	300	700	100
$\mathcal{K}_i$	$\mathcal{S}_1$	$\mathcal{S}_1$	$\mathcal{S}_2$	$\mathcal{S}_2$	$\mathcal{S}_1$
$v_{i,1}$	750	500			100
$v_{i,2}$			300	700	
$\eta_{i,1}$	2	2			1
$\eta_{i,2}$			1	1	

TABLE 3: Number of vehicles to be evacuated from buildings to shelters

FIGURE 8: Evacuation model input

algorithm using CPLEX at the first time slot is given in the table 6. Moreover the evacuation scheduling of buildings $\{\mathcal{B}_2, \mathcal{B}_1, \mathcal{B}_4, \mathcal{B}_5, \mathcal{B}_3\}$ according to this approach is illustrated by the figure 9.

	$\hat{\Phi}_{s,\mathcal{M}_{1,1}}$	$\hat{\Phi}_{s,\mathcal{M}_{1,2}}$	$\hat{\Phi}_{s,\mathcal{M}_{2,3}}$	$\hat{\Phi}_{s,\mathcal{M}_{1,4}}$	$\hat{\Phi}_{s,\mathcal{M}_{2,5}}$
$\{\mathcal{M}_{1,2}\}$	-	500 ✓	-	-	-
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1}\}$	700 ✗	500	-	-	-
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4}\}$	550	500	-	100 ✗	-
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4},\mathcal{M}_{2,5}\}$	550	500	-	0	600 ✓
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4},\mathcal{M}_{2,5},\mathcal{M}_{2,3}\}$	550	500	300 ✗	0	600
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4},\mathcal{M}_{2,5},\mathcal{M}_{2,3}\}$	550	500	0	0	600

TABLE 6: Behavior of the lexicographic maximum algorithm using CPLEX at the first time slot

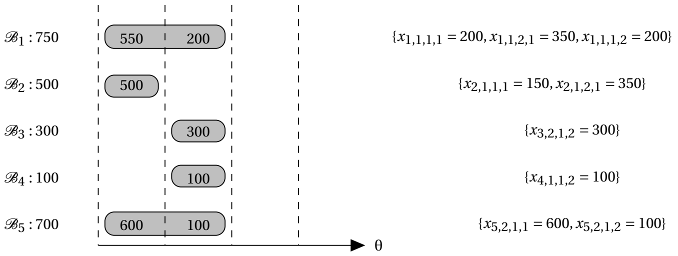

FIGURE 9: Evacuation scheduling according to lexicographic maximum flow algorithm coupled with CPLEX

In connection with the foregoing, the figure 10 shows the evacuation of an unknown building by an illegitimate path, i.e. not belonging to the set of minimal paths predetermined. This scenario could be identified, because several arcs belonging to the unique minimal path of this building have zero flow. If some cases of non-compliance with paths can be detected by this way, however a non-zero flow is not necessarily synonymous of respect of minimum paths. Hence the need to use a verification process based on the the third constraint of linear program (see previously).

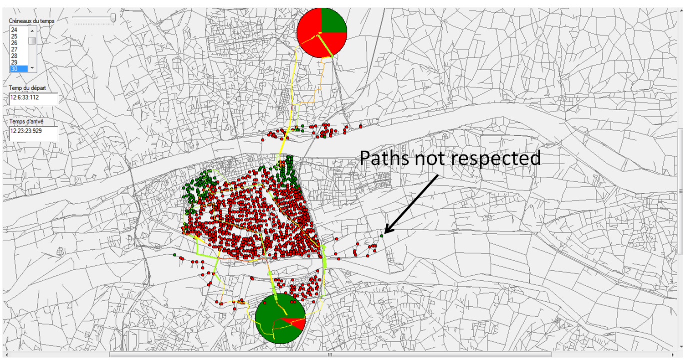

FIGURE 10: Predetermined evacuation routes not respected

On the other side, the application of the lexicographic maximum flow algorithm coupled with CPLEX have ensured the use of only the routes predetermined for each building. The figure 11 illustrates this success for the unknown building.

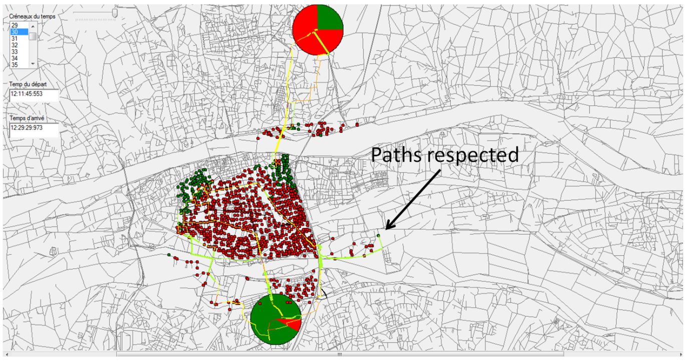

FIGURE 11: Predetermined evacuation routes respected

The assignment of flow in the evacuation network is subjected to the destinations capacities and the evacuation paths, two important elements forming the basis for the construction of evacuation plan. In other words, these two elements that have been predetermined according to several important criteria such as vulnerability, evacuees behavior, capacity, transit time, etc.) must be observed. A comparison in terms of number of slots and of number of vehicles evacuated at each time slot will be shown on a real graph in the last section.

6. New constrained Flow-Maximization algorithm

The execution time of the lexicographic maximum flow algorithm coupled with CPLEX, as we mentioned previously, is less than that of the same algorithm using push-relabel. However it's not possible to generate or update an evacuation plan in real-time using this approach and especially when the input data (buildings, paths, shelters) is almost huge. Hence the necessity for developing an efficient heuristic combining to some extent between the speed in execution and the maximization of flow in network.

In this section, we develop an heuristic method characterized by a predictive strategy for maximizing flow in network. We distinguish here between two definitions of paths : firstly the private path that does not share any arc with all other paths in the network, and secondly, the public path that has at least one common arc with one or more paths in the network.

The principle of this heuristic is based, at each iteration, on the estimation of the effects of choice of a path from the current building to be evacuated on the paths of lower priority buildings. This estimation is based on three elements : common arcs, residual number of vehicles per building and residual capacity of each path of lower-priority buildings.

The purpose of this predictive strategy is to force buildings to use first the private routes, then the public ones in a gradual manner according to the degree of estimated effect.

The predictive heuristic is presented by the algorithm 2. At each time slot (line 4, algorithm 2), the algorithm treats the set Υ of buildings not yet completely evacuated (line 5, algorithm 2). Initially, $\Upsilon = \{\mathcal{B}_1, \ldots, \mathcal{B}_n\}$. At each iteration in each time slot, the algorithm removes the priority building $\mathcal{B}_i$ from Υ, then checks if this building has more than one path (line 8, algorithm 2). If this is not the case, it gives a null effect to the unique path of $\mathcal{B}_i$ since this path is the only alternative for removing vehicles from this building. In case of multiple paths, it estimates the use effect of each path $\mathrm{A}rc^{k}_{i,j}$ of $\mathscr{B}_i$ on each path $\mathrm{A}rc^{k'}_{i',j'}$ of each lower priority building $\mathscr{B}_{i'}$, $\forall \mathscr{B}_{i'} \in \Upsilon$, $\beta_{i',j'} = 1$, $\forall k' \in \{1,\ldots,\eta_{i',j'}\}$. This estimation is based both on the nonzero residual capacity $\hat{\phi}_l - \phi^{\theta}_l \neq 0$ of common arcs of paths $\mathrm{A}rc^{k}_{i,j}$ and $\mathrm{A}rc^{k'}_{i',j'}$, and on the maximum number of vehicles $\hat{\phi}^{k'}_{\mathcal{M}_{j',i'},\mathscr{B}_{i'}}$ that may be evacuated through $\mathrm{A}rc^{k'}_{i',j'}$. Finally the use effect of route $\mathrm{A}rc^{k}_{i,j}$ on lower priority buildings takes the maximum value of :

$$\begin{cases} \dfrac{\hat{\phi}^{k'}_{\mathcal{M}_{j',i'},\mathscr{B}_{i'}}}{\hat{\phi}_l - \phi^{\theta}_l} \\ \\ \forall l \in \mathrm{A}rc^{k}_{i,j} \;\;/\;\; \hat{\phi}_l - \phi^{\theta}_l = \mathrm{Min}_{\hat{\phi}-\phi}\mathrm{A}rc^{k}_{i,j} \;\;\text{et}\;\; \hat{\phi}_l - \phi^{\theta}_l \neq 0 \\ \forall \mathscr{B}_{i'} \in \Upsilon \\ \beta_{i',j'} = 1 \\ \forall k' \in \{1,\ldots,\eta_{i',j'}\} \\ \alpha_{i',j',k',l} = 1 \end{cases}$$

After estimating the use effect of each path from $\mathscr{B}_i$ on the remaining buildings (lower priority), the algorithm gradually assigns the flow on the paths of $\mathscr{B}_i$ (lines 21-32, algorithm 2) : from least to most influential (line 22).

Algorithm 2 Predictive heuristic of maximization of flow

1: **procedure** PREDICTIVE MAXIMIZATION OF FLOW
2: $\theta = 0$
3: **repeat**
4: $\theta = \theta + 1$ ▷ openning of a new slot
5: $\Upsilon = \{\mathscr{B}_{i^*},\ldots,\mathscr{B}_{n^*}\}$ ▷ a set of buildings not yet completely evacuated
6: **for each** $\mathscr{B}_i \in \Upsilon$ **do**
7: $\Upsilon = \Upsilon \setminus \{\mathscr{B}_i\}$ ▷ Remove $\mathscr{B}_i$ from Υ
8: **if** $\eta_{i,j} > 1$ **then** ▷ $\mathscr{S}_j \in \mathbf{S}/\beta_{i,j} = 1$
9: **for** $k = 1$ to $\eta_{i,j}$ **do** ▷ If number of paths is greater than 1
10: $\rho = 0$ ▷ Effect initialized to zero
11: **for each** $l \in \mathrm{A}rc^{k}_{i,j} / \hat{\phi}_l - \phi^{\theta}_l = \mathrm{M}in_{\hat{\phi}-\phi}\mathrm{A}rc^{k}_{i,j}$ **do** ▷ minimal arcs
12: **if** $\hat{\phi}_l - \phi^{\theta}_l \neq 0$ **then** ▷ If the residual capacity is nonzero
13: $\rho = \mathrm{M}ax\{\rho, \frac{\hat{\phi}^{k'}_{\mathcal{M}_{j',i'},\mathscr{B}_{i'}}}{\hat{\phi}_l - \phi^{\theta}_l} \cdot \alpha_{i',j',k',l}\}, \forall \;\mathscr{B}_{i'} \in \Upsilon, \beta_{i',j'} = 1, \forall k'$
14: **end if**
15: **end for**
16: $\mathcal{F}[k] = \rho$ ▷ $\mathcal{F}$: Influence
17: **end for**
18: **else**
19: $\mathcal{F}[\eta_{i,j}] = 0$ ▷ If the number of paths is equal to 1
20: **end if**
21: **for** $k = 1$ to $\eta_{i,j}$ **do**
22: $\mathcal{F}[k'] \equiv \mathrm{M}in\{\mathcal{F}[1],\ldots,\mathcal{F}[\eta_{i,j}]\}$ ▷ Take the road less influential
23: $\mathcal{F}[k'] = \mathrm{M}ax\{\mathcal{F}[1],\ldots,\mathcal{F}[\eta_{i,j}]\} + 1$
24: $r^* = r_{i,j}$
25: **if** $\mathrm{Min}_{\hat{\phi}-\phi}(\mathrm{A}rc^{k'}_{i,j})$-$r_{i,j} \geq 0$ **then** ▷ Assignment of flow
26: $r_{i,j} = 0$
27: **else**
28: $r_{i,j} = r_{i,j}$- $\mathrm{Min}_{\hat{\phi}-\phi}(\mathrm{A}rc^{k'}_{i,j})$
29: **end if**sec11
30: **for** $l \in \mathrm{A}rc^{k'}_{i,j}$ **do**
31: $\phi^{\theta}_l = \phi^{\theta}_l + r^* - r_{i,j}$
32: **end for**
33: **end for**
34: **end for**
35: **until** $\sum_{i=i'}^{n'} r_{i,j} = 0$ ▷ All buildings are totally evacuated
36: **end procedure**

The application of the predictive heuristic to the graph in the figure 7 generates an assignment of flow to arcs for each time slot. At first slot, the computation of the choice effect of first path from $\mathscr{B}_2$ on firstly, the first path of $\mathscr{B}_1$, and secondly, the unique path of $\mathscr{B}_4$ is respectively given by the two tables 7 and 8. Recall that $\{\mathscr{B}_2,\mathscr{B}_1,\mathscr{B}_4,\mathscr{B}_5,\mathscr{B}_3\}$ is the established priorities list.

The effect choice of first and second path of $\mathscr{B}_2$ on the paths of lower priority buildings is respectively given by : $\mathcal{F}[1] = 1$ et $\mathcal{F}[2] = 0$. This latter is null because there is no common arc between the second path from $\mathscr{B}_2$ and the paths of lower priority buildings. Beside, the first path of $\mathscr{B}_2$ shares on the one hand, the road links a_2 and a_3 of minimum residual capacity with the first path from $\mathscr{B}_1$ and, on the other hand, the arc a_3 of minimum residual capacity with the unique path of $\mathscr{B}_4$.

This computation shows the predictive aspect of this heuristic forcing $\mathscr{B}_2$ to first use its second way, and then the first one. This allows for $\mathscr{B}_1$ to maximize its flow in this first time slot (see figure 9).

The abandonment of the predictive strategy in this heuristic converts this latter to a greedy algorithm i.e. choice effect of every path is considered null (not computed) and then the assignment of flow from each building $\mathscr{B}_i$ is performed by order of paths $\mathscr{K}_{i,j}$ connecting $\mathscr{B}_i$ to $\mathscr{S}_j$. Accordingly, the execution time of the resulting greedy algorithm is much shorter than that of the predictive heuristic. A comparison in terms of number of slots and number of vehicles evacuated at each time slot will be carried out in the next section.

l	a_2	a_3
$\frac{\hat{\phi}^{1}_{\mathcal{M}_{1,1},\mathscr{B}_1}}{\hat{\phi}_l - \phi_l}$	$\frac{350}{350-0}$	$\frac{350}{350-0}$

TABLE 7: Estimation of choice effect of $\mathrm{A}rc^{1}_{2,1}$ on $\mathrm{A}rc^{1}_{1,1}$

l	a_3
$\frac{\hat{\phi}^{1}_{\mathcal{M}_{1,4},\mathscr{B}_4}}{\hat{\phi}_l - \phi_l}$	$\frac{100}{350-0}$

TABLE 8: Estimation of choice effect of $\mathrm{A}rc^{1}_{2,1}$ on $\mathrm{A}rc^{1}_{4,1}$

The table 9 shows the assigned flow from each building by the predictive heuristic at the first time slot.

	$\phi_{s,\mathcal{M}_{1,1}}$	$\phi_{s,\mathcal{M}_{1,2}}$	$\phi_{s,\mathcal{M}_{2,3}}$	$\phi_{s,\mathcal{M}_{1,4}}$	$\phi_{s,\mathcal{M}_{2,5}}$
$\{\mathcal{M}_{1,2}\}$	-	500	-	-	-
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1}\}$	550	500	-	-	-
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4}\}$	550	500	-	0	-
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4},\mathcal{M}_{2,5}\}$	550	500	-	0	600
$\{\mathcal{M}_{1,2},\mathcal{M}_{1,1},\mathcal{M}_{1,4},\mathcal{M}_{2,5},\mathcal{M}_{2,3}\}$	550	500	0	0	600

TABLE 9: Predictive heuristic Behavior at the first time slot

The successive allocations by the predictive heuristic at different time slots results in the evacuation scheduling of buildings illustrated in the figure 12.

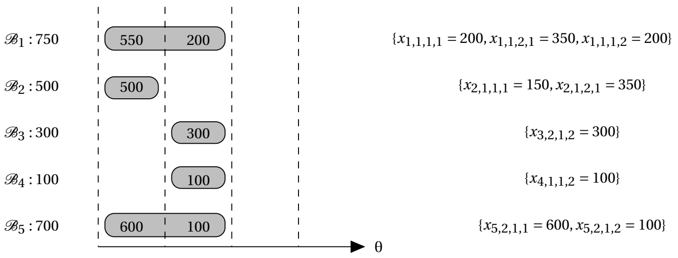

FIGURE 12: Scheduling evacuation applying the predictive heuristic

7. Applications and results

The study site on which the model is applied and tested is the valley of Tours [5] (see [4]). The elaborated evacuation network corresponds to the 3-best paths between buildings and shelters (see [3]) and is characterized by : 13446 nodes, 22676 arcs, 9825 buildings grouped in 2753 buildings [6] and 2 safety points. The figure 13 shows this area and the buildings to be evacuated to two shelters (ZRO [7]), one in the North and other in the South. For reasons of clarity of the figures, we perform the tests here only on the center of this valley : city of Tours. The graph corresponding to this city is characterized by : 6754 nodes, 11270 arcs, 3975 buildings grouped in 1308 buildings and 2 shelters. The slot time unit is equal to 15 minutes and the evacuation priorities list is established according to several criteria (hazard level, dikes, etc.).

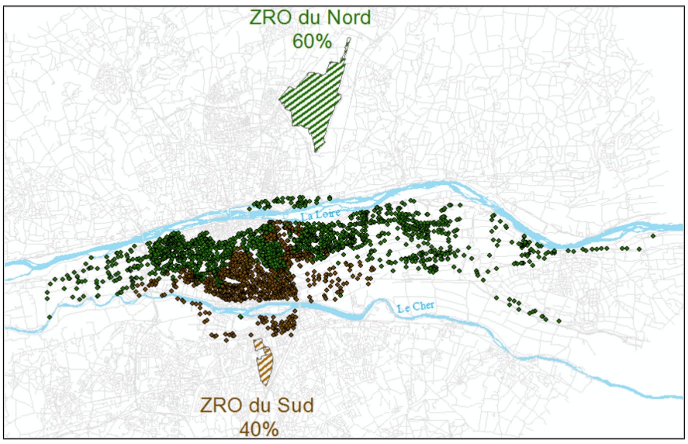

FIGURE 13: The valley of Tours, FR 37

The non-compliance, to some extent, with evacuation paths in push-relabel constitutes the first serious problem because the importance to respecting these paths carefully predetermined. A second serious problem resulting from the first one is the discontinuity evacuation of some buildings i.e. a building is evacuated on several slots spaced in time. We represent respectively the continuity and the discontinuity of flow from buildings by blue and red lines. For example, if a building is evacuated at slots 1 and 3, then we draw a blue line between the slots 1 and 2, another red between the 2^{nd} and the 3^{rd}, and finally a blue line between the third and fourth slot.

5. France 37
6. By assigning each building to the nearest network node
7. Zone de Regroupement et d'Orientation

The next two figures 14 and 15 show successively the parallel evacuation of buildings to two shelters (North and South) and the continuity/discontinuity of flow from buildings according to the lexicographic maximum flow algorithm coupled with push-relabel.

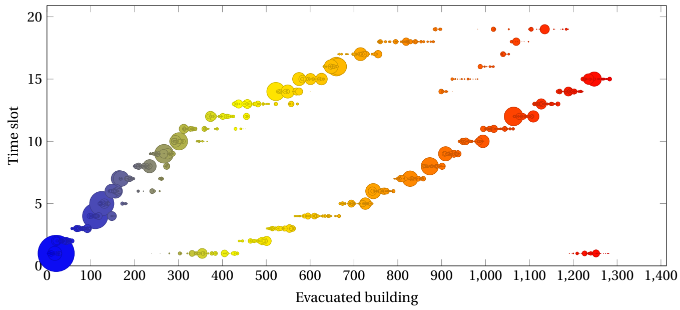

FIGURE 14: Evacuation carried out by the lexicographic maximum flow approach coupled with push-relabel

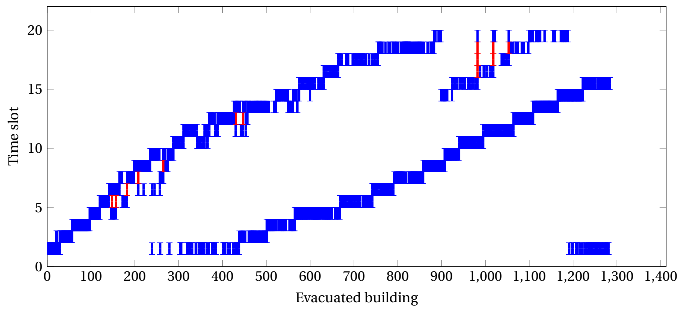

FIGURE 15: Discontinuity of flow from some buildings : lexicographic maximum flow approach using push-relabel

The discontinuity of flow from buildings is due to the fact that evacuation paths are not observed i.e. push-relabel build a new paths in order to maximize the flow. We couple the lexicographic maximum flow algorithm with a integer linear program (CPLEX) enabling to maximize flow under the constraint of respect of paths. The resulting new approach allows also to evaluate the efficiency of the predictive heuristic in the maximization of flow in network.

The figures 16 and 17 illustrate the absence of discontinuity of flow from buildings using the lexicographic maximum flow algorithm coupled with CPLEX. In other words, they show the importance of compliance with predetermined evacuation paths in order to ensure a full flow continuity from buildings during the evacuation.

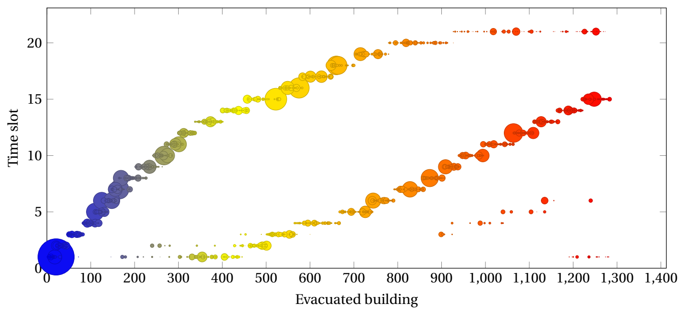

FIGURE 16: Evacuation carried out by the lexicographic maximum flow algorithm coupled with CPLEX

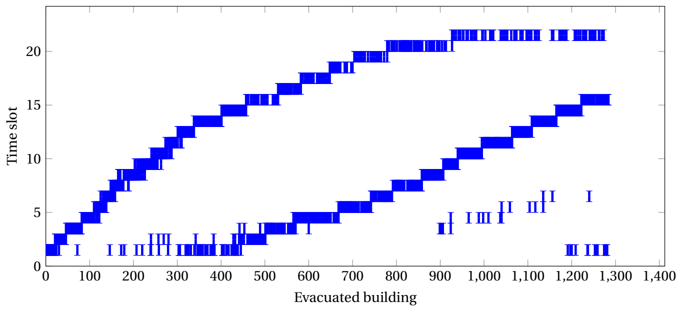

FIGURE 17: Continuity of flows from each building : lexicographic maximum flow algorithm coupled with CPLEX

Finally, and since the predictive heuristic ensures the respect of priorities and paths, its evaluation will be performed only according to the number of evacuation time slots.

The following figure 18 shows the number of vehicles evacuated by time slots according to the following approach : lexicographic maximum flow algorithm (Push-relabel), lexicographic maximum flow algorithm (CPLEX), predictive heuristic and greedy heuristic.

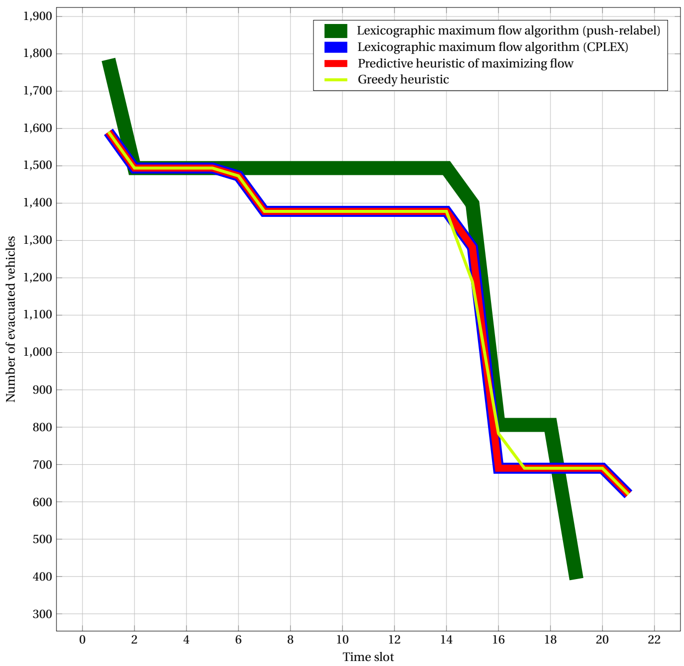

FIGURE 18: Comparison of the number of vehicles evacuated per slots between the different approachs developped

The number of slots varies depending on the algorithm used. Note that the number of remaining vehicles to be evacuated from each building is the same in all the algorithms at first time slot. This number may differ from second slot since it depends on the assignment of flow carried out at first time slot and which may in turn be different depending on the applied algorithm. At first time slot the lexicographic maximum flow algorithm coupled with push-relabel evacuates 1785 vehicles, while this same algorithm coupled with CPLEX injects 1591 vehicles in the network. This difference is due of course to the non-compliance with evacuation paths and therefore the use of the maximum load of network. The two heuristics, predictive and greedy, were able to evacuate the same number of vehicles assigned by the lexicographic maximum flow algorithm (CPLEX). This equal number indicating the efficiency of heuristics is result of existence of two elements : 1) the urban transport network is complex and 2) the density of buildings is high. In other words, an incorrect order of paths for assigning flow from current building may effects on the lexicographical maximization of flow from the next buildings but not necessarily on the full maximization of flow in the network.

Conclusion

In this paper we developed a traffic assignment model for planning a mass vehicular evacuation. Several algorithms (exact methods and heuristics) were presented and compared according to two aspects : the maximization of flow and the continuity of evacuation of buildings. We showed also the importance of traffic assignment models which take into consideration the predetermined evacuation routes relative to the flow models.

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References

[1] Alaeddine, H., Maïzia, M., Serrhini, K., and Néron, E. (2014a). A mezoscopic vehicles pursuit model for managing traffic during a massive evacuation. *portail.scd.univ-tours.fr/iii/encore/?lang=eng.*

[2] Alaeddine, H., Néron, E., Serrhini, K., and Maïzia, M. (2014b). A novel polynomial algorithm for the lexicographic maximum dynamic flow problem with several sources applied to evacuation planning. *portail.scd.univ-tours.fr/iii/encore/?lang=eng.*

[3] Alaeddine, H., Serrhini, K., Maïzia, M., and Néron, E. (2014c). Finding the k-best paths in evacuation network. *portail.scd.univ-tours.fr/iii/encore/?lang=eng.*

[4] Alaeddine, H., Serrhini, K., Maïzia, M., and Néron, E. (2014d). A spatio-temporel optimization model for the evacuation of the population exposed to natural disasters. *portail.scd.univ-tours.fr/iii/encore/?lang=eng.*

[5] Ardekani, S., Ghandehari, M., and Nepal, S. (2011). Macroscopic speed-flow models for characterization of freeway and managed lane. *The University of Texas, Arlington, USA Department of Civil Engineering.*

[6] Bliemer, M. (2001). Analytical dynamic trffic assignment with interacting user-classes. *Delft : TRAIL Research School / Delft University Press.*

[7] Boyles, S. (2011). User equilibrium and system optimum. *CE 392C Fall.*

[8] Bretschneider, S. (2012). Mathematical models for evacuation planning in urban areas. *Lecture Notes in Economics and Mathematical Systems 659.*

[9] Bretschneider, S. and Kimms, A. (2012). Pattern-based evacuation planning for urban areas. *European Journal of Operational Research*, 216 :57–69.

[10] Burkard, R., Dlaska, K., and Klinz, B. (1993). The quikest flow problem. *ZOR-Methods and Models of Operations Research*, 37 :31–58.

[11] Catell, K., Dinneen, M., and Fellows, M. (1994). A simple linear-time algorithm for finding paths-decompsitions of small width. *Department of Computer Science, University of Victoria, Victoria, British Columbia, V8W 3P6, Canada.*

[12] Fleischer, L. (2001). Universally maximum flow with piecewise-constant capacities. *Networks*, 38(3) :115–125.

[13] Fleischer, L. and Skutella, M. (2003). Quickest flows over time.

[14] Fleischer, L. and Tardos, . (1998). Efficient continuous-time dynamic network flow algorithms. *Operations Research Letters*, 23 :71–80.

[15] Ford, L. and Fulkerson, D. (1955). Maximal flow through a network. *Canadian Journal of Mathematics*, 8 :399–404.

[16] Ford, L. and Fulkerson, D. (1958). Constructing maximal dynamic flows from static flows. *Operations Research*, 6 :419–433.

[17] Ford, L. and Fulkerson, D. (1962). Flows in networks. *Princeton University Press, Princeton, NJ.*

[18] Gale, D. (1959). Transient flows in networks. *The Michigan Mathematical Journal*, 6 :59–63.

[19] Gartner, N., Messer, C., and Rathi, A. (2001). Traffic flow theory a state-of-the-art report. *http ://www.tft.pdx.edu/docs/revised_monograph_2001.pdf.*

[20] Goldberg, A. and Tarjan, R. (1988). A new approach to the maximum flow problem. *Journal of the ACM*, 35(4) :921–940.

[21] Greenshields, B. (1935). A study of highway capacity. *Proceedings Highway Research Record, Washington*, 14 :448–477.

[22] Hamacher, H. and Tjandra, S. (2001). Mathematical modelling of evacuation problems : A state of art. *Beritche des Fraunhofer Istitut Techno- und Wirtschaftsmathematik, Nr. 24.*

[23] Hamacher, H. and Tufekci, S. (1987). On the use of lexicographic min cost flows in evacuation modeling. *Department of Industrial and System Engineering, University of Florida, Gainesville, Florida 32611.*

[24] Han, L., Yuan, F., Chin, S., and Hwang, H. (2006). Global optimization of emergency evacuation assignments. *Interfaces*, 36 :pp. 502–513 issn0092–2102 eissn1526–551X 06 3606 0502.

[25] Hoppe, B. and Tardos, E. (1994). Polynomial time algorithms for some evacuation problems. *Proc. of 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, pages 433–441.

[26] Kent, P. (2008). Route choice modelling in dynamic traffic asssignment. *The Netherlands. MSc Thesis.*

[27] Kohler, E., Langkau, K., and Skutella, M. (2002). Time-expanded graphs for flow-dependent transit times. *Technische Universitat Berlin Institut fur Mathematik, Sekr. MA 6–1, Max-Planck-Institut fur Informatik Stuhlsatzenhausweg 85, 66123 Saarbr¨ucken, Germany.*

[28] Lim, G., Zangeneh, S., Baharnemati, M., and Assavapokee, T. (2012). A capacitated network flow optimization approach for short notice evacuation planning. *European Journal of Operational Research 223*, pages 234–245.

[29] Minieka, E. (1973). Maximal, lexicographic, and dynamic network flows. *Operations Research*, 21 :517–527.

[30] Nptel (2006). Traffic assignement. *CHAPTER 10. NPTEL.*

[31] Orda, A. and Rom, R. (1995). On continuous network flows. *Operations Research Letters*, 17 :27–36.

[32] Peeta, S. and Zialiaskopoulos, A. (2001). Foundations of dynamic traffic assignment : The past, the present and the future. *Networks and Spatial Economics*, 1 :pp. 233–265.

[33] Pel, A., Bleimer, M., and Hoogendoorn, S. (2011). Modelling traveller behaviour under emergency evacuation conditions. *Delft University of Technology, Delft, the Netherlands.*

[34] Qiana, Z., Shenb, W., and Zhanga, H. (2012). System-optimal dynamic traffic assignment with and without queue spillback : its path-based formulation and solution via approximate path marginal cost. *Preprint submitted to Transportation Research Part B.*

[35] Stepanov, A. and Smith, J. (2008). Multi-objective evacuation routing in transportation networks. *European Journal of Operational Research.*

[36] Trani, A. (2009). Introduction to transportation engineering. traffic flow models. *Virginia Polytechnic Institute and State University Blacksburg, Virginia Fall.*

[37] Wardrop, J. (1952). Some theoretical aspects of road traffic research. *Proceedings of the Institution of Civil Engineers, Part II*, 1(36) :352–378.

[38] Wilkinson, W. (1971). An algorithm for universal maximal dynamic flows in a network. *Operations Research*, 19 :1602–1612.